

ECE 302: Lecture 1.1 Infinite Series

Prof Stanley Chan

School of Electrical and Computer Engineering
Purdue University



Outline

- 1.1 Infinite Series
 - 1.1.1. Geometric Series
 - 1.1.2. Binomial Series
- 1.2 Approximations
 - 1.2.1. Taylor Approximation
 - 1.2.2. Exponential Series
 - 1.2.3. Logarithmic Approximation
- 1.3 Integration
 - 1.3.1. Odd and Even Functions
 - 1.3.2. Fundamental Theorem of Calculus
- 1.4 Linear Algebra (Optional)
 - 1.4.1. Inner Products (Optional)
 - 1.4.2. Matrix Calculus (Optional)
 - 1.4.3. Matrix Inversion (Optional)
- 1.5 Combinatorics
 - 1.5.1. Permutation
 - 1.5.2. Combination

Sum of **Finite** Geometric Series

Theorem

The sum of a **finite geometric series** of power n is

$$\sum_{k=0}^n r^k = 1 + r + r^2 + \dots + r^n = \frac{1 - r^{n+1}}{1 - r}. \quad (1)$$

Sum of **Infinite** Geometric Series

Corollary

Let $0 < r < 1$. The sum of an **infinite geometric series** is

$$\sum_{k=0}^{\infty} r^k = 1 + r + r^2 + \dots = \frac{1}{1-r}. \quad (2)$$

Can $r > 1$?

So here is the infinite series for $0 < r < 1$.

$$\sum_{k=0}^{\infty} r^k = 1 + r + r^2 + \dots = \frac{1}{1-r}. \quad (3)$$

What happens if $r > 1$?

What happens if $r = 0$?

Example 1

Compute the infinite series $\sum_{k=2}^{\infty} \frac{1}{2^k}$.

Derivative

Corollary

Let $0 < r < 1$. It holds that

$$\sum_{k=1}^{\infty} kr^{k-1} = 1 + 2r + 3r^2 + \dots = \frac{1}{(1-r)^2}. \quad (4)$$

Example 2

Compute the infinite sum $\sum_{k=1}^{\infty} k \cdot \frac{1}{3^k}$.

Outline

- 1.1 Infinite Series
 - 1.1.1. Geometric Series
 - 1.1.2. Binomial Series
- 1.2 Approximations
 - 1.2.1. Taylor Approximation
 - 1.2.2. Exponential Series
 - 1.2.3. Logarithmic Approximation
- 1.3 Integration
 - 1.3.1. Odd and Even Functions
 - 1.3.2. Fundamental Theorem of Calculus
- 1.4 Linear Algebra (Optional)
 - 1.4.1. Inner Products (Optional)
 - 1.4.2. Matrix Calculus (Optional)
 - 1.4.3. Matrix Inversion (Optional)
- 1.5 Combinatorics
 - 1.5.1. Permutation
 - 1.5.2. Combination

Combination: n choose k

Definition

The symbol $\binom{n}{k}$ denotes n choose k , and is defined as

$$\binom{n}{k} \stackrel{\text{def}}{=} \frac{n!}{k!(n-k)!}. \quad (5)$$

Example. Compute $\binom{5}{3}$ and $\binom{6}{2}$.

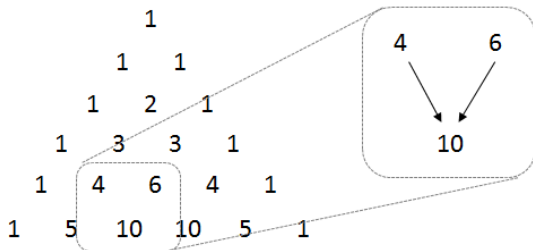
Pascal Identity

Theorem (Pascal Identity)

Let n and k be positive integers such that $k \leq n$. Then,

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}. \quad (6)$$

Proof: See note.



Binomial Series

Theorem

For any real numbers a and b , the binomial series of power n is

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k, \quad (7)$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Proof: See note.

Example. $(1 + x)^3 =$

Example

Let $0 < p < 1$. Find $\sum_{k=0}^n \binom{n}{k} p^{n-k} (1-p)^k$.

Questions?