# ECE 302: Lecture 3.5 Moment and Variance

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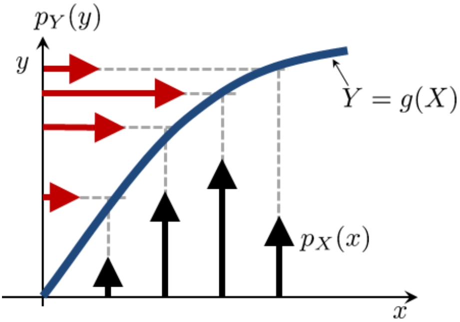
# Outline

# Properties of $\mathbb{E}[X]$

**Property (1. Function of $X$)**

*For any function $g$,*

$$\mathbb{E}[g(X)] = \sum_x g(x) p_X(x).$$



# Properties of $\mathbb{E}[X]$

Property (2. Linearity)

*For any function g and h,*

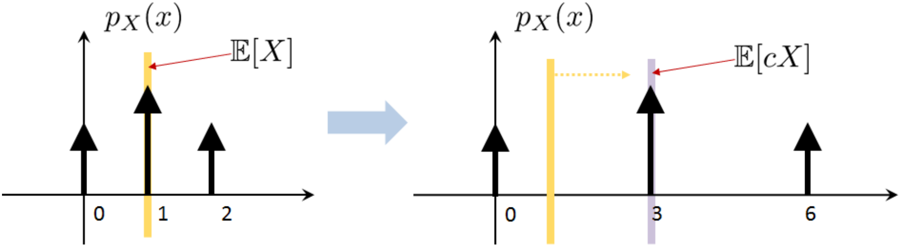
$$\mathbb{E}[g(X) + h(X)] = \mathbb{E}[g(X)] + \mathbb{E}[h(X)].$$

# Properties of $\mathbb{E}[X]$

**Property (3. Scale)**

*For any constant c,*

$$\mathbb{E}[cX] = c\mathbb{E}[X].$$

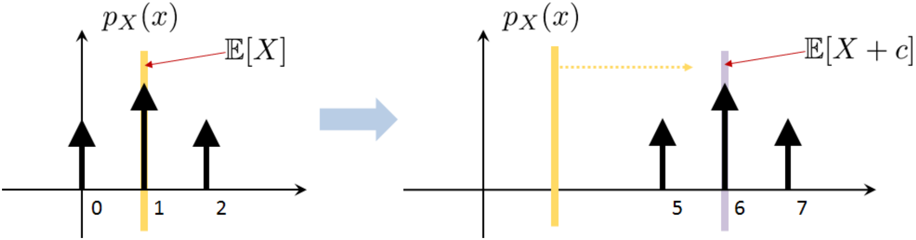


# Properties of $\mathbb{E}[X]$

**Property (4. DC Shift)**

*For any constant $c$,*

$$\mathbb{E}[X + c] = \mathbb{E}[X] + c.$$



# Outline

# Moment

### Definition

The $k$-**th moment** of a random variable $X$ is

$$\mathbb{E}[X^k] = \sum_x x^k \, p_X(x).$$

**Example**. Flip a coin 3 times. Let $X$ be the number of heads. Then,

$$p_X(0) = \frac{1}{8}, \;\; p_X(1) = \frac{3}{8}, \;\; p_X(2) = \frac{3}{8}, \;\; p_X(3) = \frac{1}{8}.$$

The second moment $\mathbb{E}[X^2]$ is

# Variance

### Definition

The **variance** of a random variable $X$ is

$$\mathrm{Var}[X] = \mathbb{E}[(X - \mu_X)^2],$$

where $\mu_X = \mathbb{E}[X]$ is the expectation of $X$. $\sqrt{\mathrm{Var}[X]}$ is called the **standard deviation**.

**Example**. $X =$ coin flip with probability $p$. Find variance of $X$.

# Properties of Variance

**Property**

*The variance of a random variable $X$ has the following properties*

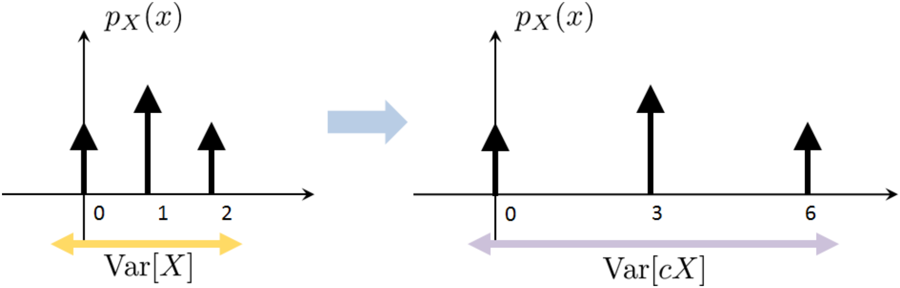
(a) **Moment**.

$$\mathrm{Var}[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2.$$

# Properties of Variance

(b) **Scale**. For any constant $c$,

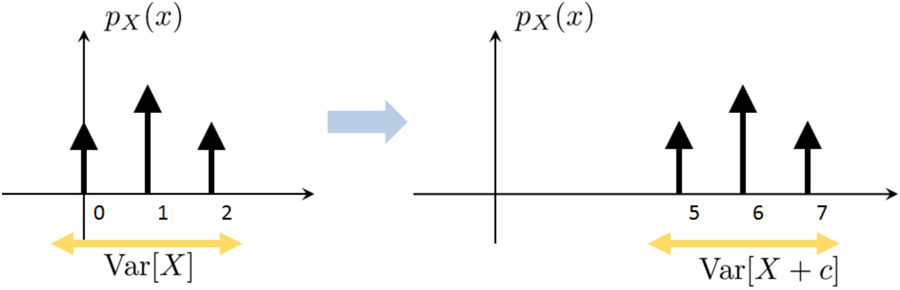
$$\mathrm{Var}[cX] = c^2 \mathrm{Var}[X].$$



# Properties of Variance

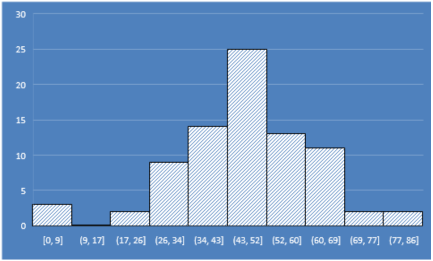
(c) **DC Shift**. For any constant $c$,

$$\mathrm{Var}[X+c] = \mathrm{Var}[X].$$



# Coming back to this problem ...

| | |
|---|---|
| Student 1 | 86.00 |
| Student 2 | 76.20 |
| Student 3 | 29.10 |
| Student 4 | 26.38 |
| Student 5 | 60.86 |
| ... | ... |
| Student 71 | 48.04 |
| Student 72 | 30.20 |
| Student 73 | 55.44 |
| Student 74 | 49.92 |
| Student 75 | 17.60 |




Add 10 points to everyone. Then,

- Will the mean change? Yes, $\mathbb{E}[X] + 10$.
- Will the standard deviation change? No. Remains $\sqrt{Var[X]}$
- If the letter grades are curved, will this change the grades? No.

**Questions?**