ECE 302: Lecture 4.5 Uniform Random Variable

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Outline

Overall schedule:

- Continuous random variables, PDF
- CDF
- Expectation
- Mean, mode, median
- Common random variables
  - **Uniform**
  - Exponential
  - Gaussian
- Transformation of random variables
- How to generate random numbers

Today's lecture:

- Definition of uniform random variables
- Properties
- Examples

Definition

Definition

Let X be a continuous uniform random variable. The PDF of X is

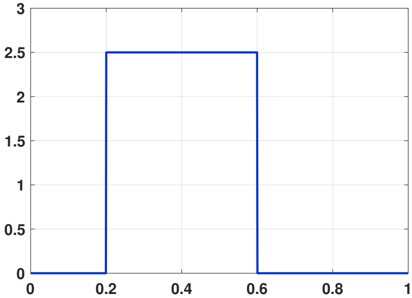
$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

where $[a, b]$ is the interval on which X is defined. We write

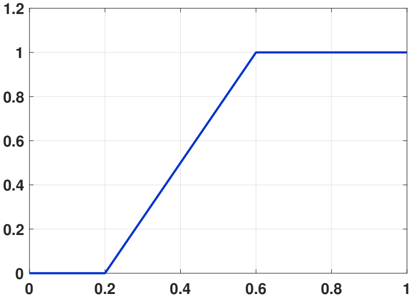
$$X \sim \text{Uniform}(a, b)$$

to say that X is drawn from a uniform distribution on an interval $[a, b]$.

PDF and CDF



(a) PDF



(b) CDF

Figure: The PDF and CDF of $X \sim \text{Uniform}(0.2, 0.6)$.

CDF

The CDF of a uniform random variable is

$$F_X(x) = \begin{cases} 0, & x < a, \\ \frac{x-a}{b-a}, & a \le x \le b, \\ 1, & x > b. \end{cases} \tag{2}$$

Computing probability

Example. Let $X \sim \text{Uniform}(-2, 3)$. Find

(a) $\mathbb{P}[0 \leq X \leq 1]$.
(b) $\mathbb{P}[X^2 \leq 2]$.

Discrete uniform random variables

If X is discrete, then

$$p_X(k) = \frac{1}{b-a+1}, \quad k = a, a+1, \ldots, b.$$

The presence of "1" in the denominator of the PMF is due to the fact that k runs from a to b, including the two end points.

Moments

Theorem

If $X \sim \mathrm{Uniform}(a, b)$, *then*

$$\mathbb{E}[X] = \frac{a+b}{2}, \quad \textit{and} \quad \mathrm{Var}[X] = \frac{(b-a)^2}{12}.$$

Function of uniform random variables

Example. Let $\Theta \sim \text{Uniform}(0, 2\pi)$. Let $X = \cos(\omega t + \Theta)$. Find $\mathbb{E}[X]$.

Summary

- When do we need uniform random variables?
  - Modeling quantization error
  - Generating random numbers according to an arbitrary PDF
- What do remember about a uniform random variable?
  - PDF:

$$f_X(x) = \frac{1}{b-a}, \quad \text{for} \quad a \leq x \leq b.$$

  - CDF:

$$F_X(x) = \frac{x-a}{b-a}, \quad \text{for} \quad a \leq x \leq b.$$

  - Expectation:

$$\mathbb{E}[g(X)] = \int_a^b g(x) f_X(x) dx$$

Questions?